## 12.1 EXERCISES

**1.** Suppose you start at the origin, move along the $x$-axis a distance of 4 units in the positive direction, and then move downward a distance of 3 units. What are the coordinates of your position?

**2.** Sketch the points $(0, 5, 2)$, $(4, 0, -1)$, $(2, 4, 6)$, and $(1, -1, 2)$ on a single set of coordinate axes.

**3.** Which of the points $P(6, 2, 3)$, $Q(-5, -1, 4)$, and $R(0, 3, 8)$ is closest to the $xz$-plane? Which point lies in the $yz$-plane?

**4.** What are the projections of the point $(2, 3, 5)$ on the $xy$-, $yz$-, and $xz$-planes? Draw a rectangular box with the origin and $(2, 3, 5)$ as opposite vertices and with its faces parallel to the coordinate planes. Label all vertices of the box. Find the length of the diagonal of the box.

**5.** Describe and sketch the surface in $\mathbb{R}^3$ represented by the equation $x + y = 2$.

**6.** (a) What does the equation $x = 4$ represent in $\mathbb{R}^2$? What does it represent in $\mathbb{R}^3$? Illustrate with sketches.
(b) What does the equation $y = 3$ represent in $\mathbb{R}^3$? What does $z = 5$ represent? What does the pair of equations $y = 3$, $z = 5$ represent? In other words, describe the set of points $(x, y, z)$ such that $y = 3$ and $z = 5$. Illustrate with a sketch.

**7–8** Find the lengths of the sides of the triangle $PQR$. Is it a right triangle? Is it an isosceles triangle?

**7.** $P(3, -2, -3)$, $Q(7, 0, 1)$, $R(1, 2, 1)$

**8.** $P(2, -1, 0)$, $Q(4, 1, 1)$, $R(4, -5, 4)$

**9.** Determine whether the points lie on straight line.
(a) $A(2, 4, 2)$, $B(3, 7, -2)$, $C(1, 3, 3)$
(b) $D(0, -5, 5)$, $E(1, -2, 4)$, $F(3, 4, 2)$

**10.** Find the distance from $(3, 7, -5)$ to each of the following.
(a) The $xy$-plane
(b) The $yz$-plane
(c) The $xz$-plane
(d) The $x$-axis
(e) The $y$-axis
(f) The $z$-axis

**11.** Find an equation of the sphere with center $(1, -4, 3)$ and radius 5. What is the intersection of this sphere with the $xz$-plane?

**12.** Find an equation of the sphere with center $(2, -6, 4)$ and radius 5. Describe its intersection with each of the coordinate planes.

**13.** Find an equation of the sphere that passes through the point $(4, 3, -1)$ and has center $(3, 8, 1)$.

**14.** Find an equation of the sphere that passes through the origin and whose center is $(1, 2, 3)$.

**15–18** Show that the equation represents a sphere, and find its center and radius.

**15.** $x^2 + y^2 + z^2 - 6x + 4y - 2z = 11$

**16.** $x^2 + y^2 + z^2 + 8x - 6y + 2z + 17 = 0$

**17.** $2x^2 + 2y^2 + 2z^2 = 8x - 24z + 1$

**18.** $4x^2 + 4y^2 + 4z^2 - 8x + 16y = 1$

**19.** (a) Prove that the midpoint of the line segment from $P_1(x_1, y_1, z_1)$ to $P_2(x_2, y_2, z_2)$ is

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}, \frac{z_1 + z_2}{2}\right)$$

(b) Find the lengths of the medians of the triangle with vertices $A(1, 2, 3)$, $B(-2, 0, 5)$, and $C(4, 1, 5)$.

**20.** Find an equation of a sphere if one of its diameters has endpoints $(2, 1, 4)$ and $(4, 3, 10)$.

**21.** Find equations of the spheres with center $(2, -3, 6)$ that touch (a) the $xy$-plane, (b) the $yz$-plane, (c) the $xz$-plane.

**22.** Find an equation of the largest sphere with center $(5, 4, 9)$ that is contained in the first octant.

**23–32** Describe in words the region of $\mathbb{R}^3$ represented by the equation or inequality.

**23.** $y = -4$

**24.** $x = 10$

**25.** $x > 3$

**26.** $y \geqslant 0$

**27.** $0 \leqslant z \leqslant 6$

**28.** $z^2 = 1$

**29.** $x^2 + y^2 + z^2 \leqslant 3$

**30.** $x = z$

**31.** $x^2 + z^2 \leqslant 9$

**32.** $x^2 + y^2 + z^2 > 2z$

**33–36** Write inequalities to describe the region.

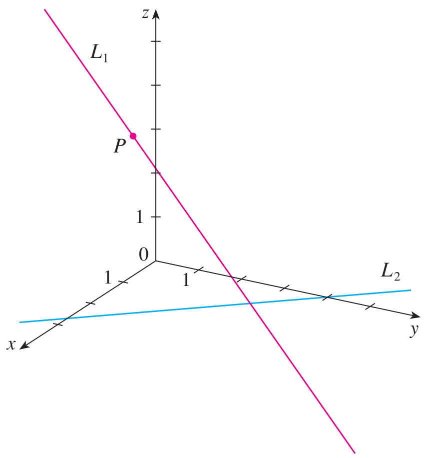
**33.** The region between the $yz$-plane and the vertical plane $x = 5$

**34.** The solid cylinder that lies on or below the plane $z = 8$ and on or above the disk in the $xy$-plane with center the origin and radius 2

**35.** The region consisting of all points between (but not on) the spheres of radius $r$ and $R$ centered at the origin, where $r < R$

**36.** The solid upper hemisphere of the sphere of radius 2 centered at the origin

**37.** The figure shows a line $L_1$ in space and a second line $L_2$, which is the projection of $L_1$ on the $xy$-plane. (In other words, the points on $L_2$ are directly beneath, or above, the points on $L_1$.)




(a) Find the coordinates of the point $P$ on the line $L_1$.

(b) Locate on the diagram the points $A$, $B$, and $C$, where the line $L_1$ intersects the $xy$-plane, the $yz$-plane, and the $xz$-plane, respectively.

**38.** Consider the points $P$ such that the distance from $P$ to $A(-1, 5, 3)$ is twice the distance from $P$ to $B(6, 2, -2)$. Show that the set of all such points is a sphere, and find its center and radius.

**39.** Find an equation of the set of all points equidistant from the points $A(-1, 5, 3)$ and $B(6, 2, -2)$. Describe the set.

**40.** Find the volume of the solid that lies inside both of the spheres

$$x^2 + y^2 + z^2 + 4x - 2y + 4z + 5 = 0$$

and

$$x^2 + y^2 + z^2 = 4$$

## 12.2 VECTORS

The term **vector** is used by scientists to indicate a quantity (such as displacement or velocity or force) that has both magnitude and direction. A vector is often represented by an arrow or a directed line segment. The length of the arrow represents the magnitude of the vector and the arrow points in the direction of the vector. We denote a vector by printing a letter in boldface ($\mathbf{v}$) or by putting an arrow above the letter ($\vec{v}$).

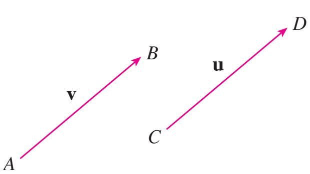



**FIGURE 1**
Equivalent vectors

For instance, suppose a particle moves along a line segment from point $A$ to point $B$. The corresponding **displacement vector** $\mathbf{v}$, shown in Figure 1, has **initial point** $A$ (the tail) and **terminal point** $B$ (the tip) and we indicate this by writing $\mathbf{v} = \overrightarrow{AB}$. Notice that the vector $\mathbf{u} = \overrightarrow{CD}$ has the same length and the same direction as $\mathbf{v}$ even though it is in a different position. We say that $\mathbf{u}$ and $\mathbf{v}$ are **equivalent** (or **equal**) and we write $\mathbf{u} = \mathbf{v}$. The **zero vector**, denoted by $\mathbf{0}$, has length 0. It is the only vector with no specific direction.

### COMBINING VECTORS

Suppose a particle moves from $A$ to $B$, so its displacement vector is $\overrightarrow{AB}$. Then the particle changes direction and moves from $B$ to $C$, with displacement vector $\overrightarrow{BC}$ as in Figure 2. The combined effect of these displacements is that the particle has moved from $A$ to $C$. The resulting displacement vector $\overrightarrow{AC}$ is called the *sum* of $\overrightarrow{AB}$ and $\overrightarrow{BC}$ and we write

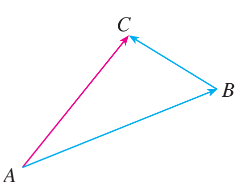



**FIGURE 2**

$$\overrightarrow{AC} = \overrightarrow{AB} + \overrightarrow{BC}$$

In general, if we start with vectors $\mathbf{u}$ and $\mathbf{v}$, we first move $\mathbf{v}$ so that its tail coincides with the tip of $\mathbf{u}$ and define the sum of $\mathbf{u}$ and $\mathbf{v}$ as follows.

**DEFINITION OF VECTOR ADDITION** If $\mathbf{u}$ and $\mathbf{v}$ are vectors positioned so the initial point of $\mathbf{v}$ is at the terminal point of $\mathbf{u}$, then the **sum** $\mathbf{u} + \mathbf{v}$ is the vector from the initial point of $\mathbf{u}$ to the terminal point of $\mathbf{v}$.